“Cardinal Number” Interpretation Guide
CHILDREN’S UNDERSTANDING OF NUMEROSITY

Background:

Young children have to master various early mathematical concepts before they can begin adding and subtracting. One of these early concepts is the “Cardinal Number Principle”: the idea that the last number counted equates to the final size of a set. At an early age, children may be able to count to high numbers, but they may not yet have mastered the Cardinal Number Principle: that is, they may not fully understand the amount that each number represents.

The “Cardinal Number” research toy is based on experiments conducted at the University of Michigan, in which researchers examined how children understand “unique numerosity” (the idea that number words refer to a specific amount of objects within a set). Researchers recruited children 2 to 4 years of age. In the first part of the study, children were given several tasks that related to counting and recognizing specific quantities (e.g. children were asked to point to a picture with five objects or give a researcher five objects). In the second part of the study, participants performed the *transform-sets* task (the basis for our museum activity), which studied what *physical* changes to a set children considered relevant to its “numerosity” (the number of items in the set). The researchers started by counting out loud and placing a certain number of toy erasers inside a box and then asking the children exactly how many were inside. Then, they would “transform” the set in one of four ways: by shaking the box, rotating the box 360 degrees, opening the box and adding one more eraser to it, or opening the box and removing one eraser from it. Then the researcher would then ask the child how many erasers were inside the box *now*. In the third part of the study, children participated in activities to help researchers determine if they could differentiate between a set with a specific number of objects and another set that is labeled as having *“a lot”* of objects. They hypothesized that children, even for numbers they could not count to, could realize that those numbers represented specific quantities.

Researchers found that children – even as young as 2 or 3 years old – *do* understand that numbers represent specific quantities. Although many were not able to count more than a couple of objects, they knew that when an object was removed from or added to a set, the previously specified number no longer represented the set. For example, if children were presented with five objects and then one object was removed, the children were more likely to say that the number of objects was any number other than five (e.g. six or four objects). This indicates that children know that number words, such as five, refer to specific quantities – even if they cannot count to five or know exactly “how many” the number word “five” represents.

Why is this important?

Math is often considered one of the most important domains that children need to master. While even very young children may learn how to count, their true understanding of numbers may come after that ability develops. This indicates that math learning involves both memorization and understanding. Research studies suggest that even without a complete understanding of numbers, children are learning about mathematical concepts, such as numerosity, entirely on

their own. A strong grasp of basic number ideas, including the Cardinal Number Principle, enables further development and acts as the building blocks of other mathematical ideas, such as addition and subtraction.

Method:

Recruiting Methods:

1. Prominently display the house and frogs to make it appealing and approachable for children.
2. Introduce yourself to parents, and explain that you are doing an activity based on a study done at the University of Michigan, where researchers looked at children's understanding of math and numbers.
3. Ask children if they would like to play a game with you.

Important Modifications

- This activity was adapted from the *transform-sets* task. The original study included several other activities that dealt with similar ideas (see *Sarnecka, 2004*). This specific task was chosen for the museum because (compared to the other tasks which were verbal-only) it had props (the box and toys), which may make the activity more appealing to a museum audience.
- In the original study, researchers had the children do each of the seven tasks at least eight times, for a total of 56 trials. In the *transform-sets* task specifically, each child performed eight trials. In the museum activity, children do three trials, in addition to the warm up.
- In the study, the researchers used erasers for their objects; the museum version uses plastic frogs, but any animal figurine could be substituted.
- In the museum activity, the simple box was changed to a house where the frogs live and a storyline was added to the activity to make it more engaging for children – this change was found helpful during prototyping.

Materials

- One toy house that can hold several small figurines, it should have a closable opening (see Appendix B for a sample design)
- One or several sets of animals (6 in each set). We used plastic frogs.
- A script for reference (see Appendix A)

Activity Instructions (the "study method")

Warm up Activity:

1. Ask the child to play a game with you (*Do you want to play a game with me?*)
2. Introduce the house where some frogs live (*This is the house near a pond where some frogs live*).
3. Show the child one frog going inside the house (*Here's a frog.* [Put the frog inside the house.]).
4. Ask the child: *What is inside the house?* Have the child answer—do not correct them.
5. Shake the house and explain that there was a storm that shook the house, but everything was okay (*There was a strong storm that night that shook the house, but everything was okay*).
6. Ask the child what is inside the house now (*What is inside the house now?*). Have the child answer—do not correct them.

Main Activity:

1. Explain that you're going to play again and ask the child to remember how many frogs are inside the house each time something happens (*Now we're going to play again except this time you're going to remember how many frogs are inside the house*).
2. Tell the child that there are five frogs spending the night at the house, and put the frogs inside, one by one (*There were five frogs that decided to spend the night at the house.* [Put frogs in house one by one]).
3. Ask the child: *How many frogs are inside the house?* Have them answer.

ROUND 1

4. Tell the child: *Another frog hopped from the pond and decided to also stay at the house.* Add a frog to the house.
5. Ask the child: *How many frogs are inside the house now?*

ROUND 2

6. Shake the house and tell the child: *There was a strong storm that night that shook the house, but everything was just fine.*
7. Ask the child: *How many frogs are inside the house now?*

ROUND 3

8. Tell the child: *Then one of the frogs inside the house left to go find some food.* Remove a frog from the house.
9. Ask the child: *Are there 4, 5, or 6 frogs in the house now?*
10. Tell the child: *Great job! Do you want to play with the frogs for a little while?* Allow the kid to play with the box and toys while you talk with the caregiver.

<u>Activity Tips</u>

Help Parents Observe

- What did your child think would change the number of frogs in the house? What did s/he think would not change the number of frogs in the house?
- Did your child remember that there were five frogs in the house initially?
- How many frogs did s/he think were in the house after it was shaken/rotated? Did s/he think that the number of frogs changed?
- How many frogs did s/he think were in the house when a frog was added/removed? Did s/he think that the number of frogs changed?

Keeping Children Interested

- Older children may have already mastered the Cardinal Number Principle, and might not understand the point of the activity. Talk to them about what the Cardinal Number Principle is, and ask whether they have younger siblings who may not understand this idea yet.
- If children seem old enough, talk with them about why the Cardinal Number Principle is important. Otherwise, allow children to play with the objects if they seem interested while you talk to their parent!

<u>Results of the Original study:</u>

- Many children (ages 2-4 years) could not generate sets of objects when given a specific number word or match a number word to its corresponding set, but still recognized that

different number words represented different sets, and sets of different sizes have different number words.

- Children were more likely to say that the number word of a set changed when an item was gained or lost than when the set was shaken or rotated. When the box was 'transformed' by rotating or shaking, children said that the number of objects inside the box was the same. When an object was removed or added, children would answer with a *different* number than what was in the box, even if that number did not accurately represent the number in the box.
- All children (ages 2-4 years) could identify quantity changes to the sets in the study, even when they couldn't count reliably to the number in the set. For example, even the children that could only reliably identify numbers up to 2 would still recognize a change to a set of 5 or 6.
- As children get older, they become more adept at understanding and using the Cardinal Number Principle. By approximately 3 ½ to 4 years of age, children begin to consistently use the Cardinal Number Principle when counting sets.

<u>Questions Parents May Ask</u>

What does this activity show?

Researchers found that children, even as young as two or three, *do* understand that number words represent specific quantities. Although many were not able to count to numbers such as five or six, they knew that when an object was removed from or added to a set, the previously specified number word no longer represented that set.

Did my child "pass"?

This is a toy we developed from a research study; it is not a test—there are no right or wrong answers! This activity is based on a completed study where researchers looked at how children think about and understand numbers—in particular, the Cardinal Number Principle (how numbers relate to objects in a set). This skill develops by about 4 years of age.

What's the Cardinal Number Principle? Is it important?

The Cardinal Number Principle is the idea that the final number counted represents the final size of a set. This may seem like a basic concept for most adults, but a lot of children struggle with this idea. They might be able to count to high numbers by memorizing (most children develop this skill quickly), but they may not understand what the number word they count to represents.

Can I teach my child the Cardinal Number Principle?

The Cardinal Number Principle is not an idea that can be directly taught to children. It is something that they develop individually over time. There are some ways that you can encourage them to further explore this concept – please see: *Where can I learn more?* (below).

When will my child understand this concept?

Most children develop this principle by around age 4 years, however it is important for children to develop and learn this principle at their own pace. If you are very concerned about the pace at which your child is developing, please consult a professional such as your pediatrician.

Are you still collecting data?

This activity was adapted from a research study done by the University of Michigan. That study is completed and has been published. They have already collected all of the data that they need.

This activity was designed as part of Living Laboratory, a program that brings child development researchers, museum educators, and caregivers together to better understand how children think, learn, and grow.

Where can I learn more?
To learn more about children's understanding of numbers and math concepts:

- *Einstein Never Used Flash Cards: How Our Children Really Learn--and Why They Need to Play More and Memorize Less* – a book by Kathy Hirsh-Pasek, Roberta M. Golinkoff, and Diane E. Eyer.
- More information about this study, as well as related child development studies, can be found on the researcher's website: http://www.cogsci.uci.edu/cogdev/Sarnecka/

Activities for Caregivers to try at the Museum

- Try finding some blocks and help your child count them. Emphasize how many blocks there are in a group (e.g. when you finish counting, say how many blocks there are in the set, such as "one, two, three…there are *three* blocks!"). You can even count the components of any exhibit (e.g., "How many buttons can we find?").
- Ask your child if there are things they think that they *can't* count, and then see if you can! For example ask if you can count how many steps a person makes, even though they leave no footprints behind.

Activities for Caregivers to Try at Home

- Try counting with your child at home! Allow your child to count a set of tangible objects (such as blocks) and ask them how many objects there are in the set. For example "One block, two blocks, three blocks, and four blocks. There are four blocks". This will help children connect the concept of counting and the total number of objects in the set.
- Play board games that involve counting the number of spaces you travel on the board!
- Try counting sets of more than three objects. Studies have found that children who count sets with more than three objects are more likely to have a better understanding of early mathematical concepts. This will deepen their understanding of smaller numbers, as well as expose them to larger numbers.

Sources and Resources:
Sarnecka, B. 2004 "Six Does Not Just Mean a Lot: Preschoolers See Number Words as Specific." *Cognition* 92(3): 329-52.

Acknowledgements:
This activity was originally developed for Living Laboratory® in the Discovery Center at the Museum of Science, Boston.

Appendix A – Activity Script

"Warm up" Activity:

- *Do you want to play a game with me?*
- *This is the house near a pond where some frogs live.* (Show the child the house.)
- *Here's a frog.* (Put the frog inside the house.)
- *What is inside the house?* (Have the child answer — **do not correct them.**)
- (Shake the house.) *There was a strong storm that night that shook the house, but everything was okay.*
- *Now what's inside the house? (Have the child answer —* ***do not correct them.****)*

Main Activity

- *Now we're going to play again except this time you're going to remember how many frogs are inside the house.*
- *There were five frogs that decided to spend the night at the house.* (Put the frogs in house one by one.)
- *How many frogs are inside the house?* (Have them answer.)

First Round**:**

- *Another frog hopped from the pond and decided to also stay at the house.* (Add a frog to the house.)
- *How many frogs are inside the house?*

Second Round:

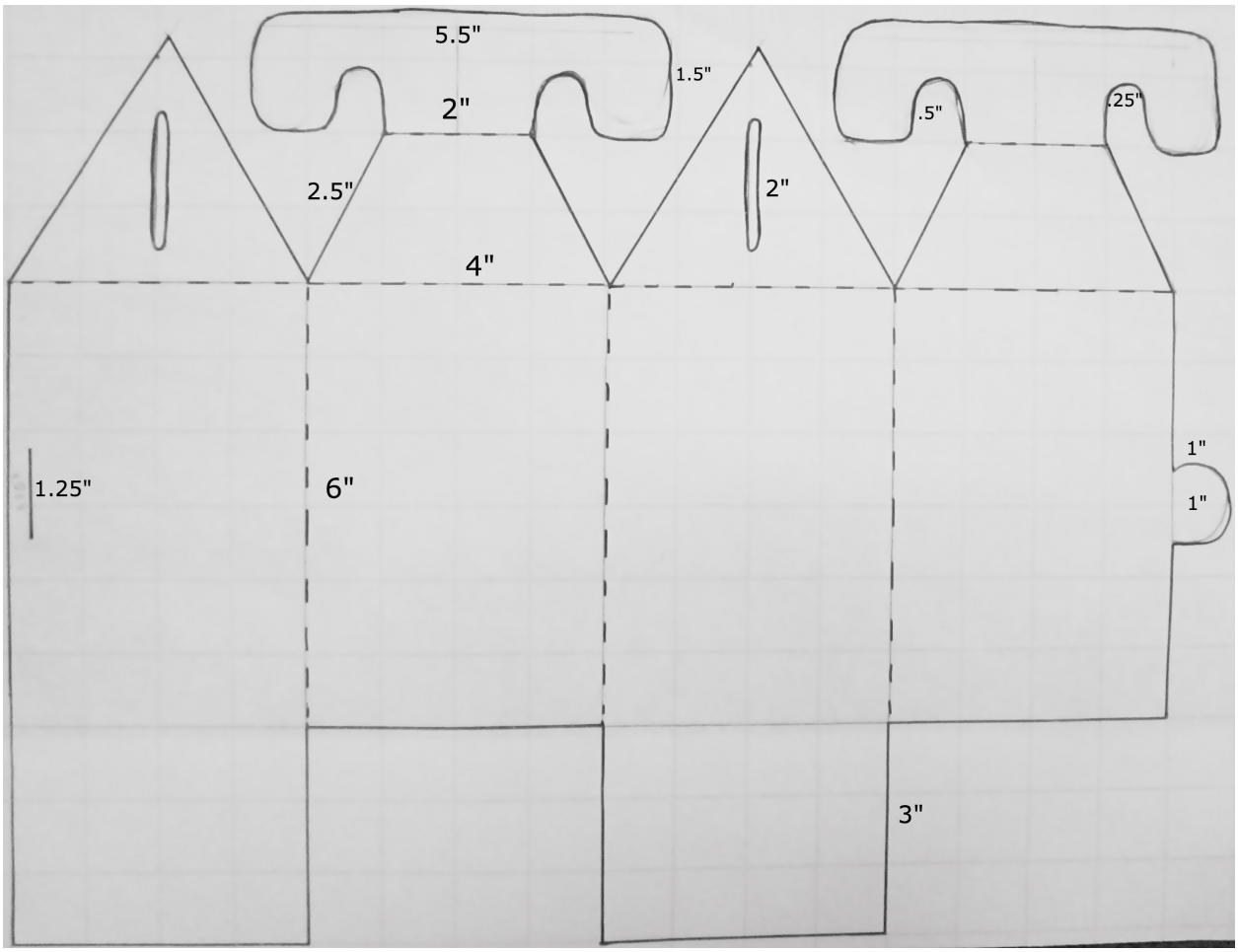
- *There was a strong storm that night that shook the house, but all the frogs were just fine.* (Shake the house.)
- *How many frogs are inside the house now?*

Third Round:

- *Then one of the frogs inside the house left to go find some food.* (Remove a frog from the house.)
- *Are there 4, 5, or 6 frogs in the house now?*
- (Allow the child to play with the box and toys while you talk with the caregiver**.)**

Appendix B – “House” Sample Design

Houses were created using the following blueprint:



Decorate the house! The house used in the Discovery Center at the Museum of Science, Boston was decorated using paints and decorative paper.

Create a closeable “door” in the house by cutting a circular hole in one side of the house and covering the hole with a moveable flap of fabric. In the version at the Museum of Science, this was done by taping a piece of fabric above the hole on the inside wall of the house.